A Combinatorial Auction for Joint Radio and Processing Resource Allocation in C-RAN

Mira Morcos, Jocelyne Elias, Fabio Martignon, Lin Chen, Tijani Chahed

***Abstract*—In this paper, we propose a truthful combinatorial auction for the joint radio and processing resource allocation problem in the context of a Cloud-based Radio Access Network (C-RAN). We formulate the auction as an Integer Linear Program (ILP), taking into accurate account interference constraints while leveraging radio resource reuse to generate an optimal revenue for the RAN operator. Then, we propose *Truthful Greedy Approach* (TGA), an effective and truthful heuristic that guarantees a close-to-optimum revenue compared to the one obtained with the ILP formulation. Extensive simulations, conducted in representative network scenarios, compare and evaluate our auction with state-of-the-art approaches from the literature, showing its effectiveness.**

Keywords: C-RAN, Multi-resource allocation, Combinatorial auction, Truthfulness.

I. Introduction

Virtualization and centralization are two key features that help the future Radio Access Networks (RAN) reduce operational and capital expenditure costs (OPEX and CAPEX, respectively) and enhance spectrum utilization [1], [2], [3]. With Cloud RAN (C-RAN), we obtain a RAN architecture composed of antennas equipped with Remote Radio Head (RRH) units connected to a centralized pool of Base Band Units (BBU). In this paper, we address the dynamic aspect of resource allocation in the context of a C-RAN, proposing a scheme that jointly allocates the spectrum and the processing units to end users. To this aim, we first propose a truthful combinatorial auction and distinguish between two types of resources: radio and processing ones. We suppose that each user requests a resource bundle by submitting a bid expressing her needs in terms of (1) number of radio resource blocks and (2) number of processing units, in addition to her valuation for obtaining both commodities. We tackle the case of an auction with single-minded bidders, where the valuation of a given user is higher than zero when she receives all the resources she requires and zero otherwise. The auction takes as input the set of users' bids and produces as outcome the allocation solution and pricing. The resource allocation and the pricing problem take into accurate account system parameters, interference constraints and resource re-usability, as well as solution constraints, namely truthfulness, individual rationality and computing complexity.

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We formulate the auction as an Integer Linear Program (ILP) generating optimal revenue. Given the fact that combinatorial auctions are computationally complex [4] and challenging in terms of truthfulness, we further propose an approach, which we term *Truthful Greedy Approach (TGA)*, that can solve the auction in polynomial time, and prove that it guarantees truthfulness. We compare the performance of the different mechanisms by running an extensive simulation campaign and show that our approach further guarantees an efficient revenue compared to the ILP formulation.

Auctions constitute an elegant business model for radio resource allocation, and gained much attention in cognitive radio networks [5], [6] as well as in Radio Access Networks [7], [8]. Combinatorial auctions, a specific type of auctions, are tailored for the allocation of multiple types of goods, where bidders compete by declaring a price for the bundle of goods they require [9]. In particular, combinatorial auctions are well adapted for virtual machine provisioning in cloud-computing systems, and proved to be efficient in generating either an efficient social welfare or a high revenue [10], [11]. Similarly, in wireless systems, combinatorial auctions are proposed as an allocation approach for spectrum, antennas and power allocation [12].

Profitable auctions should satisfy economic properties, especially truthfulness, to avoid market manipulation, and individual rationality to motivate users to participate in the auctions: truthfulness guarantees that users do not lie about their valuations by making bidding truthfully a dominant strategy, while individual rationality ensures that no user has negative profit. An efficient auction should also be computationally feasible and should guarantee an optimal revenue.

The well known Vickery-Clarke-Groves (VCG) auction generates an optimal social welfare and respects auction desired properties [8], [13]. However, VCG is shown to be NP-hard, to generate low revenues in some scenarios [14], [13], [15] and also to violate truthfulness when applied on spectrum allocation performing channel reuse [14], [10]. Accordingly, multiple approximation algorithms are proposed to solve the VCG auction in polynomial time and/or guarantee truthfulness. The authors in [8] propose a VCG-based auction for radio resource allocation in the context of C-RAN and provide a greedy algorithm to solve the auction in polynomial time. However, the greedy algorithm is not proved to be truthful. The authors in [14] propose a greedy algorithm for spectrum allocation that can perform resource reuse and guarantee truthfulness. However, simulation results showed that the auction cannot guarantee a high revenue in some scenarios.

When VCG is applied on a combinatorial auction it also loses the truthfulness property and approximation algorithms fail to guarantee this important property [16], [17], [10], [18]. The work in [17] proposes an approximation algorithm for combinatorial auctions that can guarantee truthfulness *in expectation* which guarantees that users maximize their expected profit by bidding truthfully.

Motivated by the fact that combinatorial auctions are well adapted in cloud-based networks but not much applied in the context of C-RAN, we formulate in this paper a combinatorial auction framework for joint radio and processing resource allocation. We propose an algorithm for the allocation and pricing decisions, and prove that our algorithm guarantees strong truthfulness and a high revenue with respect to existing schemes form the literature. To the best of our knowledge, our work is the first that tackles joint spectrum and processing resource allocation in a systematic manner, by means of a combinatorial auction taking into account a scenario with partial interference on spectrum resources and full interference on processing units.

The remainder of this paper is organized as follows. Section II presents the system model and the proposed auction mechanism. Section III shows the different formulations and discusses the economic properties they satisfy. Section IV illustrates and analyzes numerical results. Finally, Section V concludes the paper.

II. SYSTEM MODEL

We consider a C-RAN with a centralized BBU and distributed RRH units, leasing radio network resources in the form of bundles to a set of users. We designate by $\Omega_N = \{1, \dots, N\}$ the set of users, where N is the total number of users requiring a bundle of resources. Specifically, a *RAN bundle* is defined as a combination of spectrum and corresponding processing units. We consider the allocation of spectrum in the form of resource blocks, where a given resource block can be used by more than one user at a time if they do not interfere with each other. In total, there are R resource blocks to be allocated, and $\Omega_R = \{1, \dots, R\}$ is the corresponding set. As for the processing resources, we consider a total of P units available at the BBU level, where $\Omega_P = \{1, \dots, P\}$ is the corresponding set.

a) Radio environment: To model the radio environment, we consider $G = (V, E)$, a conflict graph where V is the set of users and E the set of edges. Two users share an edge if they interfere with each other. We deduce from G the $N \times N$ matrix A, whose element a_{kl} equals to 1 if users k and l are two interfering neighbors. We denote by $IntSet(i)$ the set of users interfering with user i.

b) Auction agents and bidding language: The auctioneer is the RAN operator, as the owner of the spectrum license and the physical pool of resources at the BBU level. The users requiring access to the network participate in the auction by submitting a bid $b_i = (d_i, q_i, w_i)$, where d_i and q_i are the number of resource blocks and the number of processing units required by user i, respectively, and w_i is the valuation declared to the auctioneer, which is equal to the maximum price user i is willing to pay in order to purchase d_i and q_i. Parameter w_i is public and might be different from the true valuation v_i, which is private to the user; we have $w_i \leq v_i$.

We denote by $u_i(b_i)$ the utility of user i, and adopt a quasi-linear utility function, widely used in auction design: $u_i(b_i) = v_i - p_i$ when user i wins and 0 otherwise; p_i is the price user i is going to pay when she bids b_i and wins (p_i should always satisfy $p_i \leq w_i$).

c) Auction framework and process: We design a combinatorial sealed bid auction, where bidders submit simultaneously their requests. Bidders are considered to be single minded as they are satisfied only if they receive the totality of the bundle they are requesting. The auction will take the bids as an input and perform the allocation and pricing decisions using the algorithms which we describe next.

Table I summarizes the parameters and decision variables introduced in our model.

Parameter	**Definition**
R, Ω_R	total number of resource blocks and the corresponding set
P, Ω_P	total number of processing units and the corresponding set
N, Ω_N	total number of users and the corresponding set
b_i, B	bid of user $i \in \Omega_N$ and the corresponding set
d_i	number of resource blocks required by user i
q_i	number of processing units required by user i
w_i, $\phi_i(w_i)$	valuation of user i to purchase d_i and q_i and the corresponding virtual valuation
v_i	user i's true valuation
$u_i(b_i)$	user i's utility when she bids b_i
p_i	the price user i is going to pay when she bids b_i and wins
$G = (V, E)$	conflict graph, where V is the set of users and E the set of edges.
$IntSet(i)$	set of users interfering with user i
Decision variable	**Definition**
x_i	Binary decision variable which is equal to 1 if user i is a winner, and 0 otherwise
r_i^k	Binary decision variable which is equal to 1 if the $k-th$ resource block is assigned to user i, and 0 otherwise
s_i^k	Binary decision variable which is equal to 1 if the $k-th$ processing unit is assigned to user i, and 0 otherwise

Table I: Parameters and variables definition

III. PROBLEM FORMULATION

We describe and discuss in this section our mathematical formulation for the combinatorial auction. We first formulate the auction as an ILP model (called *Non-Truthful Optimal Approach*) to solve the allocation decision paired with a VCG-style pricing scheme that generates an optimal revenue. However, knowing that VCG is no longer truthful when applied to combinatorial auctions [13], [18], we propose a greedy algorithm that solves the auction in polynomial time, guarantee truthfulness and still generates a high revenue, close to the optimum in several typical network scenarios.

A. Non-Truthful Optimal Approach (NTOA)

1) Allocation decision: We first start by formulating the allocation problem as an Integer Linear Program (Non-Truthful Optimal Approach, NTOA), where the operator maximizes his revenue while taking into consideration capacity constraints and performing radio resource reuse. We assume that a processing resource cannot be used by more than one user at a time.

We define the decision variables as follows:

- x_i equals 1 when bidder i wins the auction and 0 otherwise, with $i \in \Omega_N$;
- r_i^k equals 1 when the $k-th$ resource block is assigned to user i and 0 otherwise, with $k \in \Omega_R$;
- s_i^k equals 1 when the $k-th$ processing unit is assigned to user i and 0 otherwise, with $k \in \Omega_P$.

To guarantee revenue maximization, we adopt a Bayesian optimal mechanism where we consider that user i's valuation, w_i, is drawn from a given distribution $F(w_i)$, known to the auctioneer. We also consider that users bid truthfully, i.e., $w_i = v_i$. According to Myerson's theorem, described in [15], in Bayesian settings the expected revenue is equal to the sum of the virtual valuations, where a virtual valuation $\phi_i(w_i)$ is defined as follows:

$$\phi_i(w_i) = w_i - \frac{1 - F(w_i)}{f(w_i)}$$

f is the probability density function, and $\phi_i(w_i)$ is monotone non-decreasing in w_i.

Based on this, we maximize the expected revenue as follows:

$$\max \quad \sum_{i \in \Omega_N} \phi_i(w_i) x_i \tag{1}$$

s.t.

$$\sum_{j \in \Omega_N} s_j^k \quad \leq 1 \quad \forall k \in \Omega_P \tag{2}$$

$$\sum_{k \in \Omega_P} \sum_{j \in \Omega_N} s_j^k \quad \leq P \tag{3}$$

$$r_i^k + r_j^k \quad \leq 1 \qquad \forall j \in IntSet(i), \quad \forall i \in \Omega_N, \forall k \in \Omega_R \tag{4}$$

$$\sum_{k \in \Omega_R} r_i^k = d_i x_i \qquad \forall i \in \Omega_N \tag{5}$$

$$\sum_{k \in \Omega_P} s_i^k = q_i x_i \qquad \forall i \in \Omega_N \tag{6}$$

$$x_i \in \{0, 1\} \quad \forall i \in \Omega_N \tag{7}$$

$$r_i^k \in \{0, 1\} \quad \forall i \in \Omega_N, k \in \Omega_R \tag{8}$$

$$s_i^k \in \{0, 1\} \quad \forall i \in \Omega_N, k \in \Omega_P. \tag{9}$$

The objective function in Expression (1) maximizes the operator's expected revenue. Constraints (2) and (3) ensure, respectively, that a given processing unit is allocated at most to one user at a time and that the capacity in terms of processing units is respected. Constraint (4) ensures that a given resource block is allocated to at most one user i among its interfering neighbors, $IntSet(i)$. Constraints (5) and (6) ensure that a given user i receives the totality of the radio resources and processing units requested or nothing. Finally, Constraints (7), (8) and (9) are binary constraints.

2) Pricing decision: We adopt the classical VCG pricing scheme, where a user i has to pay p_i in case of winning i.e., when $x_i = 1$ and 0 otherwise. The price p_i is defined as $p_i = \phi_i^{-1}(p'_i)$ where:

$$p'_i = \max_{j \in \{\Omega_N - i\}} \sum_{j \neq i} \phi_j(w_j) x_j - \max_{j \in \Omega_N} \sum_{j \neq i} \phi_j(w_j) x_j.$$

B. Truthful Greedy Approximation (TGA)

Given that the Non-Truthful Optimal Approach (NTOA) presented above is NP-hard [19], we propose hereafter a *Truthful Greedy Approach (TGA)*, a greedy algorithm that generates a sub-optimal, yet good revenue with respect to NTOA.

We start by defining the notation we will use in the following:

- β_i: is a weight calculated for a given user i in terms of her bid's parameters as follows:

$$\beta_i = \frac{\phi_i(w_i)}{d_i |IntSet(i)| + q_i P}$$

- $pos_i(b_i, b_{-i})$: is the position of user i in the sorted list of weights, when she bids b_i; $pos_i(b_i, b_{-i})$ is decreasing in β_i.
- $avPU_i(b_i)$: is the amount of available processing units, i.e. that have not been allocated to the set of users having their weight value higher than user i's weight when she bids b_i.
- $avRB_i(b_i)$: is the set of resource blocks that have not been allocated to user i's conflicting neighbors when she bids b_i, (i.e., $\forall j \in IntSet(i)$ such that $pos_j(b_j, b_{-j}) > pos_i(b_i, b_{-i})$).

TGA consists of two procedures: an allocation decision and a pricing one.

1) Allocation decision: The allocation process is detailed in Algorithm 1: we start by calculating the weight for each user i (lines 4, 5 and 6). Then, we sort the list of users in non-increasing order with respect to the weights (line 7). In lines 8 and 9 we make sure that the allocation scheme starts with the user having the highest weight. We check in line 10 if the virtual valuation is positive and whether there are enough resource blocks available in the set $avRB_i(b_i)$ as well as enough processing units $avPU_i(b_i)$. If yes, the user is designated as winner (line 11) and assigned the requested resource blocks and processing units (lines 12-17). Finally, line 18 updates the list of available resource blocks for conflicting neighbors and the amount of available processing units.

2) Pricing decision: The idea behind the pricing decision is to charge user i the price p_i defined as:

$$p_i = \phi_i^{-1}(p'_i)$$

where $p'_i = c_l(i)(d_i |IntSet(i)| + q_i P)$ and $c_l(i) = \beta_l$ is the critical weight corresponding to the critical user l defined as follows.

Definition 3.1: For a fixed set of bids B, a critical user l is the user that, by winning, would disqualify user i for one of the following reasons: $avPU_i(b_i) - q_l < q_i$ or $|avRB_i(b_i)| - d_l <$

Algorithm 1 Truthful Greedy Approach (TGA): Allocation decision

1: **Input:** N, P, R, B,
2: **Output:** x_i, r_i^k, s_i^k, $\forall k \in \Omega_N$, $k \in \Omega_P$, $k \in \Omega_R$
3: **Init:** $avPU_i(b_i)$, $IntSet(i)$, $avRB_i(b_i)$
4: **for** $i = 1$ **to** N
5: $\quad \beta_i = \frac{\phi_i(w_i)}{d_i|IntSet(i)|+q_iP}$
6: **end**
7: Sort the set of bids B in decreasing order according to the weight β_i; let L be the sorted list and I the list of corresponding user indexes.
8: **for** $j = 1 : N$
9: $\quad i = I(j)$
10: $\quad$ **if** $\beta_i \geq 0$ & $avPU_i(b_i) \geq q_i$ & $|avRB_i(b_i)| \geq d_i$
11: $\quad\quad x_i \leftarrow 1$
12: $\quad\quad$ **for** $l = 1 : d_i$
13: $\quad\quad\quad r_i^{|avRB_i(b_i)|-l+1} = 1$
14: $\quad\quad$ **end**
15: $\quad\quad$ **for** $t = 1 : q_i$
16: $\quad\quad\quad s_i^{avPU_i(b_i)-t+1} = 1$
17: $\quad\quad$ **end**
18: $\quad\quad$ **Update** $avPU_{i'}(b_{i'})\ \forall i' \in \Omega_N$ & $avRB_k(b_k)\ \forall k \in IntSet(i)$
19: $\quad$ **else**
20: $\quad\quad x_i \leftarrow 0$
21: $\quad$ **end**
22: **end**

Algorithm 2 Truthful Greedy Approach (TGA): Pricing decision

Input: N, P, R, B,
Output: p_i
for $i = 1 : N$
$\quad$ **if** $x_i = 1$ remove b_i from B and run the allocation decision algorithm. After each iteration (line 18 in Algorithm 1):
$\quad\quad$ **if** $avPU_i(b_i) < q_i$ *or* $|avRB_i(b_i)| < d_i$.
$\quad\quad\quad c_l(i) = \beta_l$ *(with l the corresponding user index)*
$\quad\quad\quad$ **break** from Algorithm 1
$\quad\quad$ **else**
$\quad\quad\quad$ **continue** with Algorithm 1
$\quad\quad$ **end**
$\quad\quad p_i = \phi_i^{-1}(c_l(i)(d_i|IntSet(i)|+q_iP)$
$\quad$ **else**
$\quad\quad p_i = 0$
$\quad$ **end**
end

d_i. β_l verifies the following condition: if $\beta_i < \beta_l$ user i loses, while he wins otherwise.

The pricing algorithm is detailed in Algorithm 2. To determine the price p_i, we find the critical user as follows: we first remove user i's weight from the list and apply the allocation decision algorithm (Algorithm 1). After each iteration (line 18 in Algorithm 1), we verify if there are enough processing units for user i and enough resource blocks in $avRB_i(b_i)$. If yes, we continue with the next user in the list $B - \{i\}$ (Line 8 in Algorithm 1). If no, then this user is the critical user; let l be the corresponding user index and β_l the corresponding weight. We have $c_l(i) = \beta_l$ and

$$p_i = c_l(i)(d_i|IntSet(i)|+q_iP).$$

3) Auction properties: We discuss hereafter the auction properties of the TGA mechanism. We first prove that TGA is individually rational, then prove that TGA is also truthful.

a) Individual rationality: Individual rationality guarantees that no user has a negative utility, where user i's utility is defined as $u_i(b_i) = v_i - p_i$ in case of winning, and 0 otherwise.

Lemma 3.1: TGA is individually rational.
The complete proof is provided in Appendix A.

b) Truthfulness: Truthfulness, a crucial property in auction market, is challenging when it comes to combinatorial auctions. In order to guarantee truthfulness, two main properties should be satisfied: (1) the allocation must be monotone and (2) the users should pay the critical price. However, in combinatorial auctions finding the critical price can be complex [10], which makes them hard to guarantee truthfulness. With TGA, we propose an allocation mechanism that is monotonic and charges the user the critical price, which makes it truthful.

Lemma 3.2: TGA is truthful in terms of the valuations and the request in terms of resource blocks and processing units. The complete proof is provided in Appendix B.

IV. Performance evaluation

We now evaluate numerically the performance of the proposed approach, TGA, quantifying the revenue obtained and comparing it to the optimal one achieved with NTOA. We also compare the revenue of TGA with the one obtained by two additional benchmark allocation approaches in the literature: (1) the Truthful in Expectation Approach (TEA) proposed in [16], [17], [10] and (2) the Fixed Price Approach (FPA) described next.

TEA is based on the formulation described in [17] which is itself based on an ILP relaxation. It can be implemented by considering all decision variables to be fractional in $[0, 1]$. To apply this approach, we implement the allocation decision described in Algorithm 3. As for the pricing algorithm, we conduct a binary search on $\lambda \in [0, v_i]$ and run the allocation decision algorithm at each iteration: user i will pay the critical price $p_i = \lambda_0$, where λ_0 would result in $x_i = 0$ when running the allocation decision algorithm with $v_i = \lambda_0$.

As of FPA, the idea of this allocation scheme, also used in [10], is to fix a price p_0 and charge it to all winners: the allocation decision algorithm first sorts the users according to their valuation, then designates a given user i as a winner if (1) $v_i \geq p_0$ and if (2) $avPU_i(b_i) \geq q_i$ and $|avRB_i(b_i)| \geq d_i$. Regarding the pricing scheme, users pay p_0 if they win and 0 otherwise.

We specifically implemented all these different approaches using MATLAB. In particular, the ILP-based optimization model used in the NTOA approach was solved using the

Algorithm 3 Truthful in Expectation Approach (TEA): Allocation decision algorithm

Input: N, P, R, B,
Output: x_i
$R \leftarrow (1-\epsilon)R$
$P \leftarrow (1-\epsilon)P$
Solve the relaxation of the ILP model described in section III-A and let $x^* = \{x_1^*, .., x_N^*\}$ be the solution
for $i = 1 : N$
  generate randomly $y \in [0,1]$
  if $y < x_i^*$ & $|avRB_i(b_i)| < d_i$ & $avPU_i(b_i) < q_i$
    $x_i \leftarrow 1$
    Assign resources (lines 12-17 in Algorithm 1)
    Update $avPU_l(b_l) \forall l \in \Omega_N$ and $avRB_j(b_j)$ $\forall j \in IntSet(i)$
  else
    $x_i \leftarrow 0$
  end
end

CPLEX commercial solver on a server equipped with an Intel CPU at 2.60 GHz and 64 GByte of RAM.

A. Network settings

We consider a scenario where the RAN operator runs an auction over R resource blocks and P processing units. In total, N users participate in the auction. The interference graph $G = (V, E)$ is generated in a random fashion. Table II summarizes the parameter settings for the case studies we have used. We assume that users valuations are generated from a uniform distribution in $[0, 1]$. In this case, $p_0 = 0.5$ is the reserve price in TGA, i.e., it is the minimum bid valuation required to become a winner. We used this price as the fixed price in FPA.

We compare the performance of the different approaches by measuring the achieved revenue, rejection rate, social welfare and computing time as a function of the number of users, which are illustrated in Figures (1a), (1b), (1c) and (2), respectively.

B. Results and discussion

We observe in Figures (1a), (1b), (1c) that the performance of TGA is remarkably close to the ILP-based one (NTOA), in terms of revenue, social welfare and rejection rate. In fact, the revenue is only $7,5\%$ lower than the optimum, in the worst case, and just 3% lower on average. TEA is designed to generate an optimal social revenue while guaranteeing truthfulness in expectation. In fact, the authors in [17] compared TEA to VERITAS [14], a truthful approximation algorithm used for spectrum auction with channel reuse, and TEA was shown to generate higher social welfare than VERITAS. However, both approaches were shown to be weak at revenue generation. Figures (1a) and (1c) show that TGA generates higher social welfare than TEA, as well as a higher revenue: the social welfare and revenue are, respectively, 8% and 38% higher with respect to TEA.

Parameter	Values
R	10
P	20
N	$[10, 20, 30, 40, 50]$
d_i & q_i	Generated from a uniform distribution in $[1, 5]$
w_i	Generated from a uniform distribution in $[0, 1]$
$\phi_i(w_i)$	$\phi_i(w_i) = 2w_i - 1$

Table II: Parameter settings

Comparing the performance of FPA with the auction based approaches (TGA and NTOA) permits to underline the efficiency of auction mechanisms in generating higher revenue and admitting a higher number of users; this is due to the fact that auctions admit in the system users who value the commodities the most. However, Figure (1a), shows that FPA can generate higher revenue than TEA; this is due to the fact that FPA charges the users the reserve price, which is the minimum valuation that users should bid in order to win, and in this way it guarantees a threshold for the revenue achieved by the auctioneer.

As of the computational efficiency, Figure (2) shows the average computing time for the ILP-based optimal solution and for the two heuristics (TGA and TEA). We observe that TGA contributes to an average time saving of about 88% with respect to TEA, which is indeed remarkable, especially when the number of users is large.

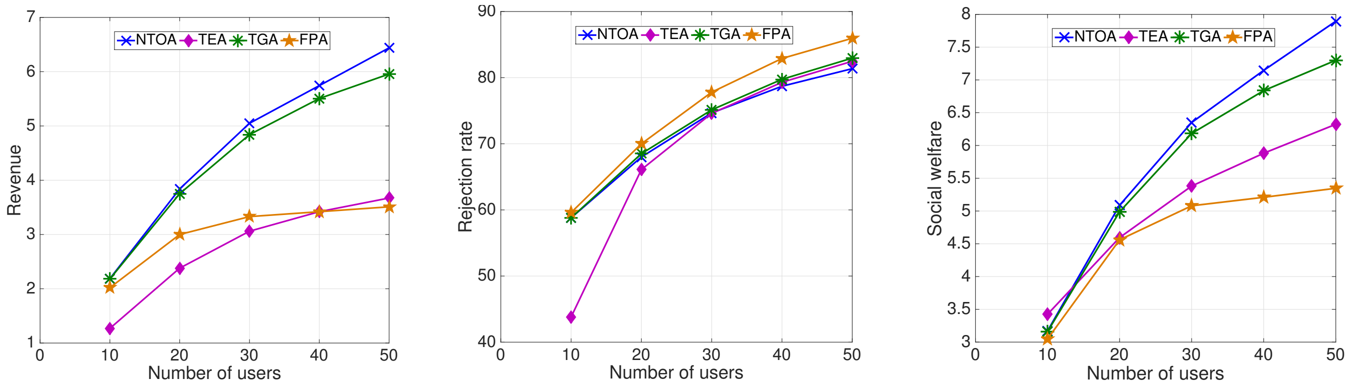
V. Conclusion

We considered in this paper a combinatorial auction for joint radio and processing allocation in the context of C-RAN. We formulated the auction as an Integer Linear Program (ILP), taking into accurate account interference constraints while leveraging radio resource reuse to generate an optimal revenue. Since solving such ILP problem can be time consuming in medium-to-large network scenarios, we further proposed Truthful Greedy Approach (TGA), an effective and truthful heuristic that guarantees a close-to-optimum revenue compared to the one obtained with the ILP formulation.

We proved TGA to be truthful and to generate a high revenue, not far from the optimal one obtained with the ILP-based solution (NTOA). Our numerical evaluation conducted in several typical network scenarios demonstrated the efficiency of TGA compared to existing, state-of-the-art heuristics from the literature.

Appendix A
Proof of lemma 3.1

We now prove that user i's utility when she bids truthfully and wins is always positive, i.e., $u(b_i) = v_i - p_i \geq 0$, $\forall i \in \Omega_N$, with $b_i = (v_i, d_i, q_i)$. Let β_l be the critical weight, we have:



(a) Revenue (b) Rejection rate (c) Social welfare

Figure 1: Performance evaluation of the proposed allocation schemes (NTOA, TGA) and comparison with state-of-the-art approaches (TEA, FPA).

$$\beta_l = \frac{\phi_l(w_l)}{d_l|IntSet(l)|+q_l P} < \beta_i = \frac{\phi_i(v_i)}{d_i|IntSet(i)|+q_i P}$$

$$=> \quad \beta_l(d_i|IntSet(i)|+q_i P) < \phi_i(v_i)$$

$$=> \quad p'_i < \phi_i(v_i)$$

$$=> \quad \phi_i^{-1}(p'_i) < v_i$$

$$=> \quad u_i(b_i) = v_i - \phi_i^{-1}(p'_i) > 0$$

which completes the proof.

Appendix B
Proof of Lemma 3.2

Truthfulness is guaranteed if bidding the truthful bid is a dominant strategy i.e., if users will not maximize their utility by bidding other than their truthful bid. It is known that an auction where the allocation is monotonic and where users pay the critical price in case of winning is a truthful auction [15], [20].

Lemma B.1: For a fixed set of bids B excluding user $i's$ bid b_i, $|avRB_i(b_i)|$ and $avPU_i(b_i)$ are decreasing with respect to $pos(b_i, b_{-i})$ and non-decreasing with respect to β_i;

i.e., for $pos_i(b'_i, b_{-i}) > pos_i(b_i, b_{-i})$ we have $avPU_i(b'_i) < avPU_i(b_i)$ and $|avRB_i(b'_i)| < |avRB_i(b_i)|$.

Proof B.1: Let us denote by J and J' the sets of users having their weights value higher than user i's weight, when she bids b_i and b'_i respectively. We have $J = \{j\} \in \Omega_N$ such that $pos_j(b_j, b_{-j}) < pos_i(b_i, b_{-i})$.

By definition, we have: $|avRB_i(b_i)| = N - \sum_{j \in \{J \cap IntSet(i)\}} d_j$ and $avPU_i(b_i) = P - \sum_{j \in J} q_j$. With $pos_i(b_i, b_{-i}) < pos_i(b'_i, b_{-i})$ we have $|J| < |J'|$ leading to, $\sum_{j \in \{J \cap IntSet(i)\}} d_j < \sum_{j \in \{J' \cap IntSet(i)\}} d_j$ and also to, $|avRB_i(b_i)| > |avRB_i(b'_i)|$ and $avPU_i(b_i) > avPU_i(b'_i)$

Lemma B.2: The allocation decision performed by TGA is monotonic.

Proof B.2: Let us consider that user i wins by bidding $b_i = (v_i, d_i, q_i)$. We demonstrate in the following that the allocation is monotonic by proving that if user i wins by bidding $b_i = (w_i, d_i, q_i)$ she will also win when bidding $b'_i = (w'_i, d_i, q_i)$ with $w'_i > w_i$ and also $b'_i = (w_i, d'_i, q'_i)$ with $d'_i < d_i$ or/and $q'_i < q_i$.

- **Case 1:** if user i wins by bidding $b_i = (v_i, d_i, q_i)$ she will also win when bidding $b'_i = (w'_i, d_i, q_i)$ with $w'_i > v_i$. In this case, given that $IntSet(i)$ and P are fixed parameters, we will have $\beta_i(b'_i) > \beta_i(b_i)$, leading to $pos_i(b'_i, b_{-i}) \leq pos_i(b_i, b_{-i})$ which, according to lemma B.1, leads to $avPU_i(b'_i) \geq avPU_i(b_i)$ and $|avRB_i(b'_i)| \geq |avRB_i(b_i)|$, which makes b'_i a winning bid.
- **Case 2:** if user i wins by bidding $b_i = (w_i, d_i, q_i)$ she will also win when bidding $b'_i = (w_i, d'_i, q'_i)$ with $d'_i < d_i$ or/and $q'_i < q_i$ since in this case we will have $\beta_i(b'_i) > \beta_i(b_i)$ and also $pos_i(b'_i, b_{-i}) \leq pos_i(b_i, b_{-i})$ leading to $avPU_i(b'_i) \geq avPU_i(b_i) > q_i > q'_i$ and $|avRB_i(b'_i)| \geq |avRB_i(b_i)| > d_i > d'_i$. And so b'_i is a winning bid.

Lemma B.3: The critical weight β_l of a given user i, for a fixed set of bids excluding user i's bid $B - \{b_i\}$, is non-decreasing with d_i and q_i.
We prove now that for $b_i = (w_i, d_i, q_i)$ and $b'_i = (w_i, d'_i, q'_i)$ with $d'_i > d_i$ and/or $q'_i > q_i$ we have $\beta_{l'} \geq \beta_l$.

Proof B.3: As shown in Algorithm 2, the critical weight β_l depends on the list of the sorted weights and on d_i and q_i. In fact, line 18 shows that with d'_i and/or q'_i, less iterations are needed to get to the user l' that would disqualify user i, with respect to l (corresponding the critical weight that disqualifies user i when bidding d_i and q_i). Hence, user l' has a lower position in the list and a higher or equal weight $\beta_{l'} \geq \beta_l$.

Now we will use the lemmas described earlier to prove the following key lemma:

Lemma B.4: User i will not maximize her utility by bidding a bid different from the true one.

Proof B.4: We prove that $u(b_i) \geq u(b'_i) \ \forall \ b'_i = (w_i, d'_i, q'_i) \neq b_i = (v_i, d_i, q_i)$ where v_i, d_i and q_i are user's i

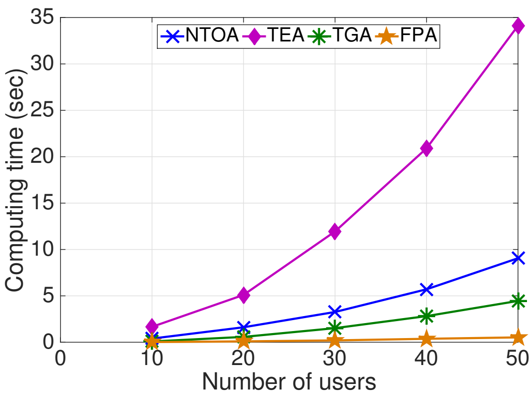


Figure 2: Computing time

truthful valuation, number of resources blocks and processing units requested, respectively. We distinguish the following 2 cases:

- **Case 1**: User i wins by bidding her truthful bid $b_i = (v_i, d_i, q_i)$. We prove now that user i will not win by bidding $b'_i = (w_i, d'_i, q'_i)$
  - if $w_i > v_i$, $d'_i = d_i$, $q'_i = q_i$: according to lemma B.2, user i will win by bidding b'_i since d_i and q_i are the truthful requirements then $c_l(b_i) = c_l(b'_i)$ (using lemma B.3) which implies $u(b'_i) = v_i - \phi_i^{-1}(p'_i) = u(b_i)$.
  - if $w_i < v_i$, $d'_i = d_i$, $q'_i = q_i$ and user i loses, we have $u(b'_i) = 0 < u(b_i)$. If user i wins, she will pay the same critical price.
  - if $w_i \neq v_i$ and $d'_i > d_i$ and/or $q'_i > q_i$: According to lemma B.3, we obtain $\beta_{l'} \geq \beta_l$ leading to $c_{l'}(b'_i) = \beta_{l'}(d'_i|IntSet(i)|+q'_i P) > c_l(b_i) = \beta_l(d_i|IntSet(i)|+q_i P)$ and $u_i(b'_i) < u_i(b_i)$.
- **Case 2**: if user i loses by bidding its truthful $b_i = (v_i, d_i, q_i)$, we have $\beta_i < \beta_l$
  - if $w_i > v_i$, in this case, if user i wins we have $u(b'_i) = v_i - \phi_i^{-1}(p'_i)$ where $p'_i = \beta_l(d_i|IntSet(i)|+q_i P) > \beta_i(d_i|IntSet(i)|+q_i P) = \phi_i(v_i)$, leading to $\phi_i^{-1}(p'_i) < v_i$ and $u(b'_i) < 0$
  - if $w_i < v_i$, then $\beta'_i < \beta_i < \beta_l$ and user i loses
  - if $w_i \neq v_i$ and $d'_i \geq d_i$ and/or $q'_i \geq q_i$, we have $\beta_i < \beta_l \leq \beta_{l'}$ (lemma B.3) leading to $p'_i = \beta_{l'}(d'_i|IntSet(i)|+q'_i P) > \beta_i(d_i|IntSet(i)|+q_i P) = \phi_i(v_i)$ leading to $\phi_i^{-1}(p'_i) > v_i$ and to $u(b'_i) < 0$.

Which completes the proof.

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